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Jensen, Jørgen Arendt

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DETECTION PROBABILITIES FOR TIME-DOMAIN VELOCITY ESTIMATION

Jørgen Arendt Jensen
Electronics Institute, build. 349
Technical University of Denmark,
DK-2800 Lyngby, Denmark

Abstract

Estimation of blood velocities by time-domain cross-correlation of successive high frequency sampled ultrasound signals is investigated. It is shown that any velocity can result from the estimator regardless of the true velocity due to the non-linear technique employed. Using a simple simulation program, it is demonstrated that the probability of correct estimation depends on the signal-to-noise ratio, transducer bandwidth, number of A-lines and number of samples used in the correlation estimate.

The influence of applying a stationary echo-canceler is explained. The echo canceling can be modeled as a filter with a transfer function depending on the actual velocity. This influences the detection probability, which gets lower at certain velocities.

An index directly reflecting the probability of detection can easily be calculated from the cross-correlation estimated. This makes it possible to assess the reliability of the velocity estimate in real time.

1 Introduction

The time-domain cross-correlation velocity estimator has received considerable attention during the last few years. Originally suggested by [1] and later rediscovered by Bonnefous [2] and others [3] for velocity estimation, it seems to improve on the currently most popular auto-correlation approach [4] in its ability to estimate velocities without aliasing. The technique also improves on the current method in its ability to utilize a standard B-scan pulse, and in correlating rf-data directly. A number of points, however, still need investigation. Stationary echo canceling has been used in a number of papers, but the consequence of its use on the estimates is probably not fully appreciated. Also the fact that the time-domain cross-correlation technique is a non-linear estimator has a number of consequences, which will be reported in this paper.

2 Basic principle

The purpose of the cross-correlation technique is to estimate the blood velocity in the direction of the ultrasound pulse propagation. This is done by emitting pulses at every T_{prf} seconds, and then acquiring the signals received. These A-lines can be written as:

$$\begin{aligned} y_i(t) &= y_{f_i}(t) + y_{s_i}(t) \\ &= y_{f_1}(t - T_{prf}(i - 1) + t_s(i - 1)) \\ &\quad + y_{s_1}(t - T_{prf}(i - 1)) \\ &= y_{f_1}(t - (i - 1)t_s) + y_{s_1}(t) \end{aligned} \quad (1)$$

y_s is the signal from stationary objects, and y_f is the signal scattered from the flowing blood. i denotes line number and t_s is the time-shift between two consecutive lines:

$$t_s = \frac{2|v| \cos(\Theta)}{c} T_{prf} \quad (2)$$

$|v|$ is the magnitude of the blood velocity, c is the propagation velocity and Θ is the angle between the flow direction and the direction of ultrasound propagation.

The amplitude of y_s can at vessel boundaries be 20 to 40 dB larger than y_f 's amplitude, so in order to estimate the low velocity flow near the walls, y_s need be removed from y . This is done by subtracting successively measured lines, so y_s cancels out as $y_{s_i}(t) = y_{s_1}(t)$. The resulting signals are then divided into segments (range gated), and subsequently cross-correlated. The lag at the position of maximum correlation then indicates the time shift between the lines, and is related to the velocity by (2).

3 Influence from stationary echo canceling

The subtraction of successive A-lines is done in order to remove the (large) stationary echoes. The processing

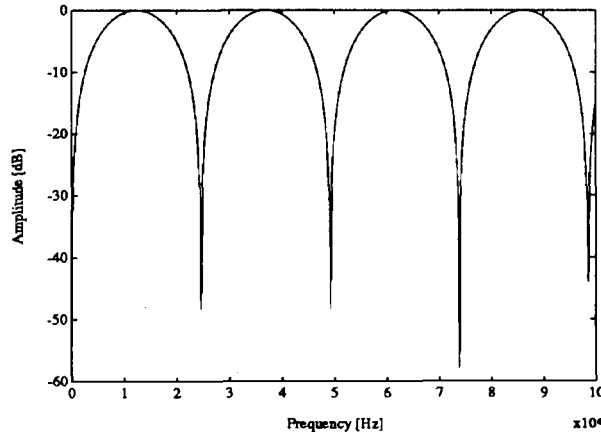


Figure 1: Transfer function of stationary echo canceling filter at $v=1$ m/s.

does, however, also influence the remaining flow signal. The subtraction can be written as:

$$e_{f_1}(t) = \frac{1}{2}(y_{f_1}(t) - y_{f_2}(t)) = \frac{1}{2}(y_{f_1}(t) - y_{f_1}(t - t_s)) \quad (3)$$

Fourier transforming the last expression yields:

$$\begin{aligned} H(f) = \frac{E_f(f)}{Y_f(f)} &= 0.5(1 - \exp(j2\pi f t_s)) \\ &= 0.5(1 - \exp(j2\pi f / f_{sh})) \\ f_{sh} &= \frac{c}{2v} f_{prf} \end{aligned} \quad (4)$$

$H(f)$ is the Fourier transform of the filter and E_f and Y_f are the transforms of e_f and y_f . The transfer function of the filter depends on the velocity of the blood. f_{sh} can be regarded as a variable sampling frequency depending on the blood velocity. The transfer function of the filter at a velocity of 1 m/s is shown in Fig. 1. The pulse repetition frequency f_{prf} was 3.2 kHz and the propagation velocity 1540 m/s. This makes f_{sh} equal to 2.46 MHz, and zeros in the transfer function at multiples of this sampling frequency are seen. The consequence of the filtration of the flow signal is a reduction in amplitude and a distortion of the pulse spectrum that depends on velocity.

The reduction in signal-to-noise ratio can be quantified by introducing a model for the signals involved. A useful model for the received signal is [5]:

$$y(t) = \int_{-\infty}^{\infty} p(t-t')s(t')dt' + n(t) = p(t)*s(t) + n(t). \quad (5)$$

$p(t)$ is the pulse echo impulse response of the ultrasound system including the electro-mechanical impulse

response of the transducer. $s(t)$ is a white, zero mean scattering signal with a Gaussian amplitude distribution. This corresponds to the scattering signal from the blood. $n(t)$ is white, zero mean noise with a Gaussian amplitude distribution. The noise is assumed independent of $s(t)$ and of the noise in the other lines acquired. The covariance of the noise is the same from line to line.

The signal-to-noise ratio is defined as:

$$\text{snr} = \sqrt{\frac{E[(p(t) * s(t))^2]}{E[n^2(t)]}} \quad (6)$$

where E is the expectation operator. The signal-to-noise ratio for the filtered signal is:

$$\begin{aligned} \text{snr} &= \sqrt{\frac{E[\{p(t) * s(t) - p(t) * s(t - t_s)\}^2]}{E[\{n_1(t) - n_2(t)\}^2]}} \\ &= \sqrt{\frac{E[\{p(t) * (s(t) - s(t - t_s))\}^2]}{E[n_1^2(t)] + E[n_2^2(t)]}} \\ &= \frac{1}{\sqrt{2}} \sqrt{\frac{E[\{p(t) * h(t; t_s) * s(t)\}^2]}{E[n^2(t)]}} \end{aligned} \quad (7)$$

$h(t; t_s)$ is the impulse response of the stationary echo canceling filter, whose response depends on the delay time t_s . From (7) it can be seen that the filtering results in a 3 dB loss in signal-to-noise ratio at high velocities, and that the loss will vary with velocity. The loss will depend on the shape of the pulse spectrum and the center frequency. An example of a pulse could be:

$$p(t) = \exp(-2(B_r f_0 \pi)^2 t^2) \cos(2\pi f_0 t) \quad (8)$$

where B_r is the relative bandwidth and f_0 the center frequency.

Using (7) and (8) the reduction in signal-to-noise ratio compared to the situation when no echo canceling is done, is calculated in [6] to be:

$$R_{\text{snr}} = \left(\frac{4\sqrt{2} + 2\exp(\frac{2}{B_r^2})}{2\sqrt{2} + \exp(\frac{2}{B_r^2}) \left[1 - \exp(-(\frac{\pi B_r f_0}{\sqrt{2} f_{sh}})^2) \right]} \right)^{1/2} \frac{-2\sqrt{2} \exp(-(\frac{\pi B_r f_0}{f_{sh}})^2) \cos(2\pi \frac{f_0}{f_{sh}})}{1} \quad (9)$$

The reduction in signal-to-noise ratio as a function of velocity is shown in Fig. 2 when using $f_0=3$ MHz, $B_r=0.2$, and $f_{prf}=3.2$ kHz. A notable reduction is seen at low velocities, where the loss can be dramatic, due to the zero in the filter when $v = 0$ m/s.

Until now only the simple subtraction filter has been investigated. More advanced filters can be employed. It

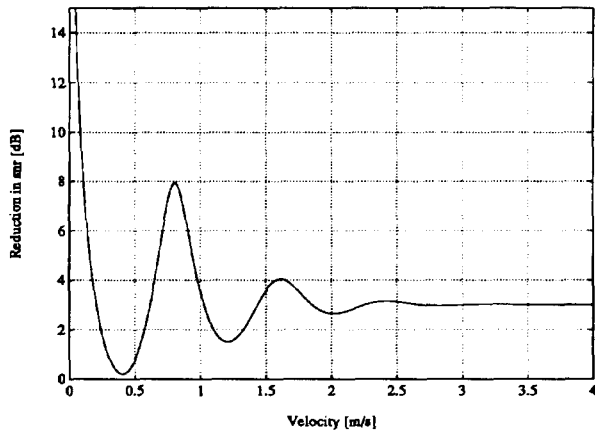


Figure 2: Loss in signal-to-noise ratio as a function of velocity.

must, however, be emphasized that the comments put forward about the simple filter still holds. A more advanced filter must also have a zero at $f = 0$, and this will introduce zeros in the transfer function at multiples of f_{sh} , which influences the signal-to-noise ratio. The influence can, however, be reduced by making the cut-off around $f = 0$ sharper.

4 Detection probabilities

The cross-correlation calculated has peaks spaced $1/f_0$ seconds apart. The height of the peaks depends on the autocorrelation of the interrogation pulse, and on the actual scattering signal $s(t)$, the size of the range gate or integration time, and on the noise in the signals entering the cross-correlation calculation. As was shown in a recent paper [5] it is quite probable that a peak other than the one at the true time displacement is the largest, so a wrong detection occurs. Due to this non-linear nature of the estimator it is only appropriate to state a probability of correct estimation as the variance bears little information concerning the estimate at low signal-to-noise ratios (below 20 dB). This can be seen from the distribution of the velocity estimates in Fig. 3. Here the frequency of results from 10000 estimates was tabulated as a function of velocity. The true velocity was 1 m/s, and a large peak (0.433) is present at this velocity. But nearly any other velocity can also result, due to the perturbation of the cross-correlation function from finite time integration and noise.

The probability of correct detection is defined as the mass between the two dotted lines in Fig. 3. It is influ-

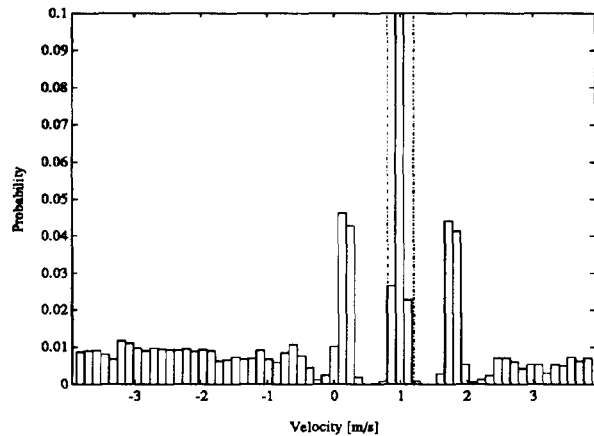


Figure 3: Distribution of velocity estimates (enlarged view). The true velocity is 1 m/s and $\text{snr} = 0$ dB. The peak at $v=1$ m/s goes to 0.433.

enced by the transducer bandwidth, the size of the range gate, or, rather, integration time (number of samples), when calculating the cross-correlation function, the number of A-lines and signal-to-noise ratio as shown in [5]. When stationary echo canceling is used, the velocity will also influence the probability as the signal-to-noise ratio then is velocity dependent as shown in the preceding section.

Lacking exact relations linking parameter variations to the probability, a simple simulation program was introduced in [5] for studying the effect of the different parameters on the probability of correct detection. The program simulates movement towards or away from the transducer. The blood scatterers are modeled as a white, zero mean random signal with a Gaussian amplitude distribution. The received signal is the scattering signal convolved with a one-dimensional transducer pulse with subsequent addition of zero mean, white, Gaussian noise. The noise is uncorrelated from line to line. The standard simulation parameters are given in table 1, and the pulse is given by (8) properly shifted in time to compensate for its non-causality.

The simulation process mimics the ideal measurement situation, in which the movement is in direction of the transducer and attenuation and diffraction are not included. It, however, has the advantage of being fast and being suited to uncover the statistical properties of the technique. It is also evident that the performance found is the absolute ideal; the estimator will not perform better in the actual measurement situation, so this establishes the upper limit on performance.

It has been suggested that use of only the sign of the

Parameter	Symbol	Value
Transducer center frequency	f_0	3.0 MHz
Relative transducer bandwidth	B_r	0.2
Sampling frequency	f_s	20 MHz
Propagation velocity	c	1540 m/s
Pulse repetition frequency	f_{prf}	3200 Hz
Samples in segment	N	32 samples
Lines for one estimate	N_{ave}	4 A-lines
Signal-to-noise ratio	snr	1
Velocity	v	1 m/s
Number of estimates	N_{trials}	10000

Table 1: Standard simulation parameters.

data acquired is sufficient in order to estimate the cross-correlation function [7]. That this is indeed possible at only a slight reduction in performance, was shown in [5]. This eases the implementation considerably making it possible to construct the electronics with a few inexpensive components and still attain real-time processing.

Fig. 4 shows the results from running the simulation program. All four cases of using full precision data or the sign and making echo canceling or not are shown. The top graph shows the marked influence from the noise, where signal-to-noise ratios below 6 dB gives rather unreliable estimates, when echo canceling is employed. A somewhat surprising result is that the curves for the full precision data does not approach one, but levels of at around 0.9 and 0.7, respectively. This is due to edge effects. There is a certain probability that strongly reflecting scatterers will enter the volume under investigation during data acquisition, and thus dominate the cross-correlation and create a false peak. The effect is most pronounced when echo canceling is done due to the elongation of the pulse from the subtraction of two time shifted pulses. The effect is not present for the one bit correlation, as this does not take amplitude into account.

A marked difference is also seen in the probability of correct detection as a function of velocity, when stationary echo canceling is used. From being relatively insensitive to variation in velocity, the probability gets strongly velocity dependent. The variation closely follows the graph given in Fig. 2, and most notable is the near zero probability of estimating velocities around 0 m/s.

The cross-correlation function estimate can be smoothed by employing a number of lines. This increases the effective integration time and thereby reduces the influence from noise, and thus increases the probability as shown in the third graph in Fig. 4.

The influence from a variation in transducer band-

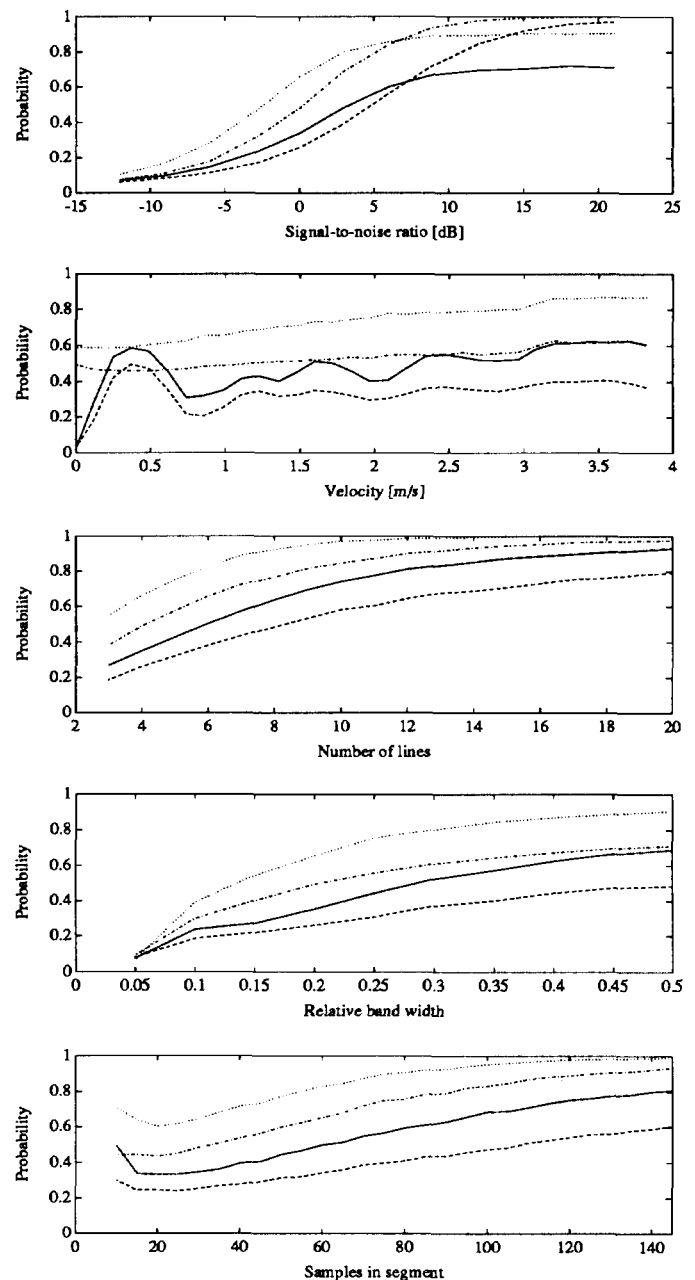


Figure 4: Variation in probability of correct detection due to different values of the parameters. — is when full precision data and echo canceling is used, --- is when using the sign and echo canceling, ... is full precision data without echo canceling, and - · - is sign data without echo canceling.

width is shown in the fourth graph. An increase in bandwidth lowers the side lobes of the autocorrelation of the pulse, and thus increases the probability. In the last graph is seen that an increase in segment length or integration time increases the probability of correct detection, when assuming a uniform velocity in the segment.

For all curves a lower probability is seen when using only the sign of the data. The reduction is, however, so small that using the sign is a viable alternative making the implementation of the technique considerably easier and cheaper.

5 A reliability index

From the last section it can be seen that the cross-correlation method does not always give correct results. Depending on the velocity and signal-to-noise ratio quite erroneous velocity estimates can result, which will appear as noise in the image. When errors occur cannot be determined *a priori* due to the dependence on noise and velocity, so we are faced with the task of devising an index that must be calculated from the actual estimate of the cross-correlation function.

A detection of an incorrect velocity occurs when one of the sidelobe peaks exceeds the main peak in the cross-correlation function, due to noise in the acquired signals. The noise introduces a spread of the energy in the cross-correlation function estimate away from the correct peak, and this spread reduces the amplitude of the peak.

The discrete cross-correlation function estimate is calculated by:

$$\hat{R}_{12}(n) = \frac{1}{N} \sum_{k=0}^{N-1} \text{sgn}[y_1(k)] \text{sgn}[y_2(k+n)] \quad (10)$$

where

$$\text{sgn}[y(k)] = \begin{cases} 1 & \text{for } y(k) \geq 0 \\ -1 & \text{for } y(k) < 0 \end{cases} \quad (11)$$

if only the sign of the data is used. N is the number of samples in one segment (range gate). If noise is neglected and y_2 is an ideal time-shifted replica of y_1 , then the peak value of $\hat{R}_{12}(n)$ becomes one, as identical sample values are multiplied. The peak value of $\hat{R}_{12}(n)$ will be less than one if y_2 is different from y_1 due to noise. With this in mind, a possible index for the reliability could be:

$$I_{rs} = \max\{\hat{R}_{12s}(n)\} \quad (12)$$

where $\max\{\}$ denotes the maximum value of the function. An index close to zero indicates a very unreliable

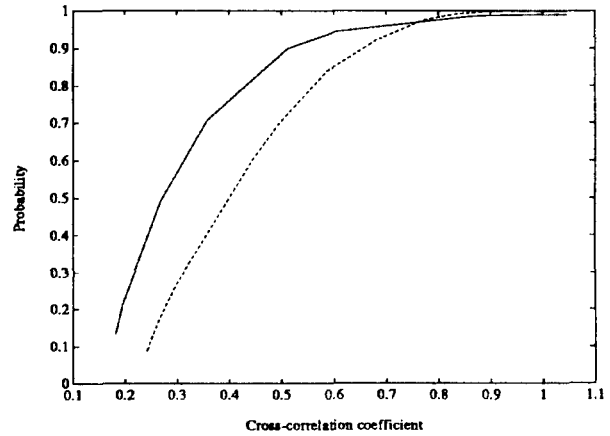


Figure 5: Relation between the maximum cross-correlation coefficient and the probability.

estimate and an index close to one indicates a very reliable estimate. The index is the cross-correlation coefficient between y_1 and y_2 , that indicates how similar the two signals are, where one indicates that they are equal.

That the averaged index is highly correlated with the probability of correct detection is shown in Fig. 5 as the dashed line. The data shown is the average for 10000 estimates at each set of parameter values. When using the full data the cross-correlation function should be divided by the power of the signals. An index based on this is shown as the solid line in Fig. 5.

From these indices a direct decision on the reliability or probability of correct detection can be made. A possible method could be to discount estimates with an index below e.g. 0.6 and then use color intensity modulation to show the reliability of the displayed velocity estimates. Another possibility is to adapt the averaging process of the correlation function estimate by employing more or less lines depending on the index to either optimize on time efficiency or reliability for the data at hand.

The curves in Fig. 5 show the averaged indices for 10000 realizations at the given parameters yielding a close correspondence between index and probability. But the individual values of the index will fluctuate around a mean value as shown in Fig. 6.

This fluctuation makes the index of little value, when a single estimate of it is used. But at a given range, the signal-to-noise ratio will be constant, so averaging can be performed. A recursive smoothing could be implemented by:

$$\hat{I}_{rm}(i) = \lambda \hat{I}_{rm}(i-1) + (1-\lambda) \hat{I}_r(i) \quad (13)$$

where $\hat{I}_r(i)$ is the current estimate of the index and \hat{I}_{rm} is

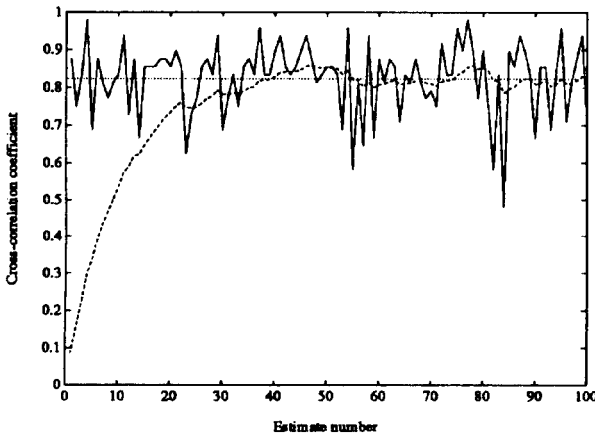


Figure 6: Reliability index $I_{r,s}$ as a function of time (— individual estimates, - - - averaged index, ... mean value).

the smoothed index. λ is the filter constant determining the time horizon of the averaging. It corresponds to a time constant of $\tau_a = 1/(1 - \lambda)$ so a value of 0.9 gives rise to averaging over roughly 10 to 20 estimates. An example of averaging with $\lambda = 0.9$ is shown as the dashed line in Fig. 6. After the initial start up phase a good estimate is found with an acceptable fluctuation around the mean value indicated by the dotted line. If the frame rate of the scanner is 20 images per second, an acceptable estimate of I_r is found after just one second.

6 Conclusion

The time-domain cross-correlation method estimates blood velocities by detecting the position of the maximum in a cross-correlation function. This is a non-linear technique, so at low signal-to-noise ratios it is only appropriate to consider the probability of correct detection. Curves were shown detailing the influence from pulse bandwidth, noise, integration time, and averaging when using a number of lines. It was also demonstrated that stationary echo canceling introduces a velocity dependent filtration of the signals involved in the cross-correlation function calculation. This leads to a velocity dependent probability of correct detection. Most notable is the near zero probability of correctly detecting velocities near 0 m/s, due to the zero at $f=0$ in the echo canceling filter.

The purpose of the filter is to remove stationary echoes emanating from vessel boundaries, so the low

blood velocities near the wall can be estimated. The probability of estimating low velocities is, however, low when using the canceling filter, making it questionable to do the stationary echo canceling. The removal of the echoes is also less of a problem compared to the standard autocorrelation technique, as a high bandwidth and thereby short pulse can be used to advantage in the time-domain method.

Due to the risk of getting wrong velocity estimates, it is beneficial to have an index reflecting the reliability of the estimates at hand. It was shown that the smoothed maximum value of the cross-correlation function calculated was in close correspondence to the probability of correct detection. This index has the advantage of taking into accounts all effects degrading the cross-correlation as noise, echo canceling, migration of scatterers in and out of the measurement volume etc., as it is calculated from the actual data. So by using this, unreliable results can be discarded and the reliability of the estimates displayed can be shown as a color intensity modulation.

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